CHAPTER	
1	**Ratio and Proportion, Indices and Logarithm**

Ratio and Proportion, Indices and Logarithm

[1] (b) Let numbers be $2x$ and $3x$.

Therefore, $(3x)^2 \quad (2x)^2 = 320$

$9x^2 \quad 4x^2 = 320$

$5x^2 = 320$

$x^2 = 64$

$x = 8$

Numbers are: $2x = 2 \times 8 = 16$

$3x = 3 \times 8 = 24$

[2] (d) As per the given information :

$$\frac{p-x^2}{q-x^2} = \frac{P^2}{q^2}$$

$q^2 (p \quad x^2) = P^2(q \quad x^2)$

$pq^2 \quad x^2 q^2 = p^2 q \quad p^2 x^2$

$x^2 (p^2 \quad q^2) = pq(p \quad q)$

$$x^2 = \frac{pq\ (p\square q)}{p^2 \square\ q^2}$$

$$x^2 = \frac{p\ q}{p+q}$$

[3] (a) Let the quantity of copper and zinc in an alloy be 9x kg and 4x kg.

Therefore, $9x = 24$

$x = \frac{24}{9} = \frac{8}{3} = 2\frac{2}{3}$ ***kg***.

So zinc = $4x = 4 \times \frac{8}{3}$ ***kg***.

= $10\ \frac{2}{3}$ ***kg***.

[4] (c) $7 \text{ Log}\left(\frac{16}{15}\right) + 5 \text{ Log}\left(\frac{25}{24}\right) + 3 \text{ Log}\left(\frac{81}{80}\right)$

= 7(log 16   log 15) + 5(log 25   log 24) +3 log (log 81   log80)

= 7 [4 log 2   (log 3 + log 5)] + 5 [2 log 5   (3 log 2 + log 3)] + 3 [4 log 3   (4log 2 + log 5)]

= 28 log 2   7 log 3   7 log 5 + 10 log 5   15 log 2   5 log 3 + 12 log 3   12 log 2   3 log 5 = log 2

[5] (c) Let the numbers be 7x and 8x .

So, $\frac{7x + 3}{8x + 3} = \frac{8}{9}$

9 (7x + 3) = 8 (8x + 3)

63x + 27 = 64x + 24

x = 3

Numbers are : 7x =7×3 =21

8x = 8× 3 =24

[6] (a) Let the number of one rupee coins be x.

Then, number of 50 paise coins is 4x

and number of 25 paise coins is 2x

So,

$x + \frac{4x}{2} + \frac{2x}{4} = 56$

4x + 8x + 2x = 56 × 4

14x = 224

$x = \frac{224}{14} = 16$

Number of 50 paise coins is 4×16 = 64

[7] (b) $(a^{1/8} + a^{-1/8})\ (a^{1/8}\quad a^{-1/8})\ (a^{1/4} + a^{-1/4})\ (a^{1/2} + a^{-1/2})$

= $(a^{1/4}\quad a^{-1/4})\ (a^{1/4} + a^{-1/4})\ (a^{1/2} + a^{-1/2})$

[using $(a^2\quad b^2) = (a\ \ b)\ (a + b)$]

= $(a^{1/2}\quad a^{-1/2})\ (a^{1/2} + a^{-1/2})$

= $a^1\quad a^{-1}$

= $a\quad \frac{1}{a}$

[8] (a) $a^{\log_a^b . \log_b^c . \log_c^d . \log_d^t}$

$a^{\frac{\log^b}{\log^a} \times \frac{\log^c}{\log^b} . \frac{\log^d}{\log^c} . \frac{\log^t}{\log^d}}$ = $\left[\text{using } \log_a b = \frac{\log^b}{\log^a}\right]$

= $a^{\frac{\log^t}{\log^a}}$

= $a^{\log_a^t}$

= t [using $a^{\log_a m} = m$]

[9] (b) Log $_{1000}$x = $\frac{1}{4}$

$(10,000)^{-1/4}$ x = [using log $a^b = x$, = a^x = b

$\frac{1}{(10,000)^{1/4}} = x$

$= \frac{1}{10} = x$

[10] (c) When number of people = 8

then, the share of each person = $\frac{1}{8}$ of the total cost.

When number of people = 7

then, the share of each person = $\frac{1}{7}$ of the total cost

Increase in the share of each person = $\frac{1}{7} - \frac{1}{8} = \frac{1}{56}$ i.e.

$\frac{1}{7}$ ***of*** $\frac{1}{8}$, i.e. $\frac{1}{7}$ of the original share of each person.

[11] (a) Let the number of coins be 3x,4x,and 5x.

Then, $3x + \frac{4x}{2} + \frac{5x}{10} = 187$

$30x + 20x + 5x = 187 \times 10$

$55x = 1870$

$x = \frac{1870}{55} = 34$

Number of coins:

One rupee = 3x = 3 ×34 = 102

50 paise = 4x = 4 × 34 = 136

10 paise =5x = 5 × 34 = 170

[12] (b) $\frac{x^{m+3n} \cdot x^{4m-9n}}{x^{6m-6n}}$

$= \frac{x^{m+3n+4m-9n}}{x^{6m-6n}}$ $\left[\text{using } \frac{x^a \cdot x^b}{x^{a+b}}\right]$

$= \frac{x^{5m-6n}}{x^{6m-6n}}$

$= x^{5m-6n-6m+6n}$ $\left[\text{using } \frac{x^a}{x^b} = x^{a-b}\right]$

$= x^{-m}$

[13] (a) Log (2a 3b) = log a log b

log (2a 3b) = log $\left(\frac{a}{b}\right)$

2a 3b = $\frac{a}{b}$

2ab $3b^2$ = a

2ab a = $3b^2$

a(2b 1) = $3b^2$

a = $\frac{3b^2}{2b□1}$

[14] (c) $\frac{1}{1+z^{a-b}+z^{a-c}} + \frac{1}{1+z^{b-c}+z^{b-a}} + \frac{1}{1+z^{c-a}+z^{c-b}}$

$= \frac{1}{1+\frac{z^{-b}}{z^{-a}}+\frac{z^{-c}}{z^{-a}}} + \frac{1}{1+\frac{z^{-c}}{z^{-b}}+\frac{z^{-a}}{z^{-b}}} + \frac{1}{1+\frac{z^{-a}}{z^{-b}}+\frac{z^{-b}}{z^{-c}}}$

$= \frac{z^{-a}}{z^{-a}+z^{-b}+z^{-c}} + \frac{z^{-b}}{z^{-b}+z^{-c}+z^{-a}} + \frac{z^{-c}}{z^{-c}+z^{-a}+z^{-b}}$

$= \frac{z^{-a}+z^{-b}+z^{-c}}{z^{-a}+z^{-b}+z^{-c}}$

= 1

[15] (d) Let the earning of A and B be 4x and 7x respectively.

New earning of A = 4x × 150% = 6x

New earning of B = 7x × 75% = 5.25

Then, $\frac{6x}{5.25x} = \frac{8}{7}$

This does not give the value of x

So, the given data is inadequate.

[16] (b) $\frac{P}{Q} = \frac{11}{12}$ and $\frac{P}{R} = \frac{9}{8}$

$\frac{P}{Q} = \frac{11\times9}{12\times9} = \frac{99}{108}$ and $\frac{P}{R} = \frac{9\times11}{8\times11} = \frac{99}{88}$

Therefore, $\frac{Q}{R} = \frac{108}{88} = \frac{27}{22}$

So, Q:R = 27:22

[17] (c) $\frac{1}{\log_{ab}^{(abc)}} + \frac{1}{\log_{bc}^{(abc)}} + \frac{1}{\log_{ca}^{(abc)}}$

$= \frac{1}{\frac{\log(abc)}{\log(ab)}} + \frac{1}{\frac{\log(abc)}{\log(bc)}} + \frac{1}{\frac{\log(abc)}{\log(ca)}}$

$\left[\text{using } \log_a b = \frac{\log b}{\log a}\right]$

$= \frac{\log(ab)}{\log(abc)} + \frac{\log(bc)}{\log(abc)} + \frac{\log(ca)}{\log(abc)}$

$= \frac{\log(ab \times bc \times ca)}{\log abc}$

$= \frac{\log a^2b^2c^2}{\log(abc)}$

$= \frac{\log(abc)^2}{\log abc} = \frac{2\log(abc)}{\log(abc)} = 2$

[18] (c) 2^{64}

= 64 log 2

= 64 × 0.30103

= 19.26592

Number of digit in 2^{64} = 20.

[19] (a) The ratio of share of A, B and C

$= \frac{1}{4} : \frac{1}{5} : \frac{1}{6}$

$= \frac{15:12:10}{60}$ = 15:12:10

Therefore, A's share = $407 \times \frac{15}{37}$ = ₹165

B's share = $407 \times \frac{12}{37}$ = ₹132

C's share = $407 \times \frac{10}{37}$ = ₹110

[20] (a) Let the income of A and B be $3x$ and $2x$ respectively and expenditures of A and B be 5y and 3y respectively.

Therefore, $3x \;\; 5y = 1500$ .... (i)

$2x \;\; 3y = 1500$ .... (ii)

Solving (i) and (ii) Simultaneously

We get $x = 3000$ and $y = 1500$

Therefore, B's income = $2x = 2 \times 3000$ = ₹ 6000

[21] (d) $4^x = 5^y = 20^z = k$ (say)

$4 = k^{1/x}$

$5 = k^{1/y}$

$20 = k^{1/z}$

$4 \times 5 = 20$

$k^{1/x} \times k^{1/y} = k^{1/z}$

$k^{1/x + 1/y} = k^{1/z}$ ($x^m \times x^n = x^{m+n}$)

$k^{\frac{x+y}{xy}} = k^{1/z}$

Therefore, $= \frac{x+y}{xy} = \frac{1}{z}$ ($x^m = x^n \quad m = n$)

$z = \frac{xy}{x+y}$

[22] (a)

$$\left(\frac{\sqrt{3}}{9}\right)^{\frac{5}{2}} \left(\frac{9}{3\sqrt{3}}\right)^{\frac{7}{2}} \times 9$$

$$= \left(\frac{3^{\frac{1}{2}}}{3^2}\right)^{\frac{5}{2}} \left(\frac{3^2}{3.3^{\frac{1}{2}}}\right)^{\frac{7}{2}} \times 3^2$$

$$= \left(3^{\frac{1}{2}-2}\right)^{\frac{5}{2}} \left(\frac{3^2}{3^{\frac{3}{2}}}\right)^{\frac{7}{2}} \times 3^2$$

$$= \left(3^{\frac{-3}{2}}\right)^{\frac{5}{2}} \quad \left(3^{\frac{2-3}{2}}\right)^{\frac{7}{2}} \times 3^2$$

$$= 3^{\frac{-15}{4}} \quad \left(3^{\frac{1}{2}}\right)^{\frac{7}{2}} \times 3^2$$

$= 3^{\frac{-15}{4}} \times 3^{\frac{7}{4}} \times 3^2$

$= 3^{\frac{-15}{4}+\frac{7}{4}+2}$

$= 3^{-2+2} = 3^0 = 1$

[23] (a) $\dfrac{\log_3^8}{\log_9^{16} \square \text{Log}_4^{10}}$

$= \log_3{}^8 . \log_{16}{}^9 . \log_{10}{}^4$

$= \log_3^{2^3} . \text{Log}_{4^2}^{3^2} . \log_{10}^{2^3}$

$= 3\log_3{}^2 \, \dfrac{2}{4} \log_2^3 \; 2 \log_{10}{}^2$

$= \dfrac{3\log 2}{\log 3} . \dfrac{1 \log 3}{2 \log 2} . \dfrac{2 \log 2}{\log 10}$

$= \dfrac{3 \log 2}{\log 10}$

$= 3 \log_{10}^2$

[24] (d) Quantity of glycerine = $40 \times \dfrac{3}{4}$ = 30 litres

Quantity of water = $40 \times \dfrac{1}{4}$ = 10 litres

Let x litres of water be added to the mixture.
Then, total quantity of mixture = $(40 + x)$ litres
total quantity of water in the mixture = $(10 + x)$ litres.

So, $\dfrac{30}{10+x} = \dfrac{2}{1}$

$30 = 20 + 2x$

$2x = 10$

$x = 5$ litres

Therefore, 5 litres of water must be added to the mixture.

[25] (d) Let the third proportional be x.

$\dfrac{a^2 \square b^2}{(a+b)^2} = \dfrac{(a+b)^2}{x}$

By cross multiplication

$x = (a+b)^2 \frac{(a+b)^2}{(a^2 - b^2)}$

$x = \frac{(a+b)^3}{(a - b)}$

[26] (c) $2^x - 2^{x-1} = 4$

$2^x - \frac{2^x}{2} = 4$

$2^x \left[1 - \frac{1}{2}\right] = 4$

$2^x \left[\frac{1}{2}\right] = 4$

$2^x = 8$

$2^x = 2^3$

$x = 3$

$x^x = 3^3$

$= 27$

[27] (a) $x = \frac{e^n - e^{-n}}{e^n + e^{-n}}$

$\frac{1}{x} = \frac{e^n + e^{-n}}{e^n - e^{-n}}$

Applying Componendo & Dividendo

$\frac{1+x}{1-x} = \frac{e^n + e^{-n} + e^n - e^{-n}}{e^n + e^{-n} - e^n + e^{-n}}$

$\frac{1+x}{1-x} = \frac{2+e^n}{2e^{-n}}$

$\frac{1+x}{1-x} = e^{2n}\ \frac{1+x}{1-x} = 2n$

$\text{Log}\left(\frac{1+x}{1-x}\right) = 2n,\ n = \frac{1}{2}\ \text{Log}\ e\left(\frac{1+x}{1-x}\right)$

[28] (b) Log 144
= Log (16 × 9) = log 16 + log9
= $\log 2^4 + \log 3^2$
= 4log2 + 2log3.

[29] (b) Let x quantity of tea worth ₹10per kg. be mixed with y quantity worth 14 per kg.
Total price of the mixture =10x +14y.
and
Total quantity of the mixture =x + y
Average price of mixture will be $\frac{10x+14y}{x+y} = 11$
10x + 14y = 11x + 11y
3y = x
$\frac{x}{y} = \frac{3}{1}$
or x : y = 3 : 1 which is the required ratio.

[30] (a) Let the present ages of persons be 5x & 7x .
Eighteen years ago, their ages = 5x 18 and 7x 18.
According to given:
$\frac{5x \square 18}{7x \square 18} = \frac{8}{13}$
65x 234 = 56x 144
9x = 90
x = 10
Their present ages are 5x = 5 × 10 = 50 years
7x = 7 ×10 = 70 years.

[31] (b) $Z = x^c$
$Z = (y^a)^c$ ($y^a = x$)
$Z = y^{ac}$
$Z = (z^b)^{ac}$ ($z^b = y$)
$Z = Z^{abc}$
abc = 1   ($x^m = x^n$ then m = n)

[32] (c) $\text{Log}_2 [\log_3(\log_2 x)] = 1$
= $\log_3 (\log_2 x) = 2^1$ (Converting into exponential form)
=$\log_2 x = 3^2$ (Converting into exponential form)
= $\log_2 x = 9$
= $x = 2^9$ (Converting into exponential form)
x = 512.

[33] (b) $\text{Log}\left(\frac{a+b}{4}\right) = \frac{1}{2}(\text{Log } a + \text{Log } b)$

$\text{Log}\left(\frac{a+b}{4}\right) = \log (ab)^{½}$

[Since, $\log_a mn = \log_a m + \log_a n$ and $n \log_a m = \log_a m^n$]

Take antilog on both sides.

$\frac{a+b}{4} = \sqrt{ab}$

$a + b = 4\sqrt{ab}$

Squaring both sides

$(a + b)^2 = (4\sqrt{ab})^2$

$a^2 + b^2 + 2\,ab = 16\,ab$

$a^2 + b^2 = 14\,ab$

$\frac{a}{b} + \frac{b}{a} = 14$, which is the required answer

[34] (a) Given : Capital invested by :

A : ₹ 126,000, B : ₹ 84,000, C: ₹ 2,10,000

The ratio of their investments is :

126 : 84 : 210 = 3 : 2 : 5

Profit (at year end) = ₹ 2,42,000 gives

A's Share = $\frac{3}{10}$ × 2,42,000 = ₹ 72,600

B's Share = $\frac{2}{10}$ × 2,42,000 = ₹ 48,400

C's Share = $\frac{5}{10}$ × 2,42,000 = ₹ 1,21,000

[35] (c) $\frac{p}{q} = -\frac{2}{3}$

So, $P = \frac{-2q}{3}$ ....(i)

Now, $\frac{2p + q}{2p - q}$

Substituting the value of p from (i)

$$\frac{2\left(\frac{-2q}{3}\right)+q}{2\left(\frac{-2q}{3}\right)-q}$$

$$\frac{\frac{-4q}{3}+q}{\frac{-4q}{3}-q}$$

$$\frac{\frac{-4q+3q}{3}}{\frac{-4q-3q}{3}}$$

$$\frac{-q}{3}\times\frac{3}{-7q}$$

$$\frac{1}{7}$$

[36] (c) Let the fourth proportional to x, 2x, (x + 1) be t, then,

$$\frac{x}{2x}=\frac{x+1}{t}$$

$$\frac{1}{2}=\frac{x+1}{t}$$

t = 2x + 2

∴ Fourth proportional to x, 2x, (x + 1) is (2x + 2)

i.e. x: 2x :: (x + 1) : (2x +2)

[37] (d) x = $3^{1/3}+3^{-1/3}$ .... (1)

On cubing both sides, we get

$$x^3=\left(3^{1/3}+3^{-1/3}\right)^3$$

$$x^3=3+3^{-1}+3\times3^{1/3}\times\frac{1}{3^{1/3}}\left(3^{1/3}+3^{-1/3}\right)$$

$$x^3=3+\frac{1}{3}+3(3^{1/3}+3^{-1/3})$$

$$x^3=3+\frac{1}{3}+3x \text{ [Using (1)]}$$

$$x^3 - 3x = \frac{9+1}{3}$$

$$3(x^3 - 3x) = 10$$

$$\therefore \quad 3x^3 - 9x = 10$$

[38] (b) $\left[1-\left\{1-(1-x^2)^{-1}\right\}^{-1}\right]^{-1/2}$

$$= \left[1-\left\{1-\frac{1}{1-x^2}\right\}^{-1}\right]^{-1/2}$$

$$= \left[1-\left\{\frac{1-x^2-1}{1-x^2}\right\}^{-1}\right]^{-1/2}$$

$$= \left[\overline{1}-\left\{\frac{-x^2}{1-x^2}\right\}^{-1}\right]^{-1/2}$$

$$= \left[1-\left\{-\frac{1-x^2}{x^2}\right\}\right]^{-1/2}$$

$$= \left[1+\frac{1-x^2}{x^2}\right]^{-1/2} = \left[\frac{x^2+1-x^2}{x^2}\right]^{-1/2}$$

$$= \left[\frac{1}{x^2}\right]^{-1/2} = \left(x^2\right)^{1/2}$$

= x

[39] (a) log (m + n) = log m + log n

log (m + n) = log (m n)   [∵ log (ab) = log a + log b]

Taking Antilog on both side

Antilog [log (m + n)] = Antilog [log mn]

∴ m + n = mn

mn – m = n

m (n   1) = n

$$m = \frac{n}{n-1}$$

[40] (a) $\text{Log}_4\ (x^2 + x) - \text{Log}_4 (x+1) = 2$

$$\text{Log}_4\left(\frac{x^2+x}{x+1}\right) = 2\left[\because \log_a m - \text{Log}_a n = \text{Log}_a\left(\frac{m}{n}\right)\right]$$

$4^2 = \frac{x^2 + x}{x+1}$

$16 = \frac{x^2 + x}{x + 1}$

$16x + 16 = x^2 + x$

$x^2 - 15x - 16 = 0$

$x^2 - 16x + x - 16 = 0$

x (x 16) + 1 (x 16) = 0

(x + 1) (x 16) = 0

x = 1 or x = 16

Since x = 1 is not possible therefore x =16

[41] (b) $\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n}$

$= \frac{2^n (1 + \frac{1}{2})}{2n (2 - 1)}$

$= \frac{\frac{3}{2}}{1} = \frac{3}{2}$

[42] (a) $2^x \times 3^y \times 5^z = 360$. ....(1)

The factors of 360 are:

$2^3 \times 3^2 \times 5$.

 $2^3 \times 3^2 \times 5^1 = 360$.............(2)

On comparing (1) and (2), we get;

$x = 3$, $y = 2$ and $z = 1$

[43] (c) $\left[\log_{10}\sqrt{25} \log_{10}(2^3) + \log_{10}(4^2)\right]^x$

= $[\log_{10} 5 - 3 \log_{10}2 + \log_{10} (2^4)]^x$

= $[\log_{10} 5 - 3 \log_{10}2 + 4 \log_{10}{}^2]^x$

= $[\log_{10} 5 + \log_{10}{}^2]^x$

= $[\log_{10} (5 \times 2)]^x$ [log (mn) = log m + log n]

= $[\log_{10} 10]^x$

= 1^x [$\log_a a = 1$]

= 1

[44] (c) Same as Ans. 26

[45] (d) $\log_a b + \log_a c = 0$

$\log_a bc = 0$

$a^0 = bc$

bc = 1

$\therefore$ $b = \frac{1}{c}$

So, b and c are reciprocals.

[46] (c) Let the number added be x

$$\frac{49+x}{68+x} = \frac{3}{4}$$

196 + 4x = 204 + 3x

x = 8

[47] (c) Let the ratio be 5x : 7x

If 10 student left, Ratio became 4 : 6

$$\frac{5x-10}{7x-10} = \frac{4}{6}$$

30x − 60 = 28x − 40

2x = 20

x = 10

$\therefore$ No. of students in each class is 5x and 7x

i.e. 50, 70

[48] (b) $2 \log x + 2 \log x^2 + 2\log x^3 + ...$

$2[\log x + \log x^2 + \log x^3 + ...]$

2[log x + 2log x + 3log x + ...]

2log x[1 + 2 + 3 .... n]

$$2 \log x \times \frac{n(n+1)}{2}$$

= n(n + 1) log x

[49] (d) 2.7777

2 + 0.7 + 0.07 + 0.007 + ...

$$2 + \left(\frac{7}{10} + \frac{7}{100} + \frac{7}{1000} + ...\right)$$

$$2 + 7\left(\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + ...\right)$$

$$2 + 7\left(\frac{1/10}{1 - 1/10}\right)$$

$$= 2 + 7 \times \frac{1}{9}$$

$= 2 + \frac{7}{9}$

$= \frac{18+7}{9}$

$= \frac{25}{9}$

[50] (a) $\left(\frac{\log_{10}x \square 3}{2}\right) + \left(\frac{11 \square \log_{10}x}{3}\right) = 2$

$3 \log_{10}x \quad 9 + 22 \quad 2 \log_{10} x = 12$

$\log_{10}x + 13 = 12$

$\log_{10}x = \quad 1$

$x = 10^{-1}$

[51] (a) $\frac{A}{B} = \frac{2}{5} = \frac{2k}{5k}$

$\frac{10A+3B}{5A+2B} = \frac{20k+15k}{10k+10k} = \frac{35k}{20k}$

$= \frac{35}{20}$

$= \frac{7}{4}$

[52] (a) Given : n = M ! for M 2

$\frac{1}{\log_2{}^n} + \frac{1}{\log_3{}^n} + \frac{1}{\log_4{}^n} + ... + \frac{1}{\log_m{}^n}$

or, $= \log_n{}^2 + \log_n{}^3 + \log_n{}^4 + ... + \log_n{}^m$ $\left(\therefore \log_b{}^a = \frac{1}{\log_a{}^b}\right)$

$= \log_n (2 \times 3 \times 4 \times .... \times m)$ $\left(\therefore \log^{(mn)} = \log^m + \log^n\right)$

$= \log_n (m!)$

$= \log_n{}^n$

$= 1$

[53] (a) Given : A : B = B : C

$B^2 = A \times C$

or $B = \sqrt{A \times C}$

& A = 1,60,000 ; C = 2,50,000

$B = \sqrt{1,60,000 \times 2,50,000}$

$B = 2,00,000$

[54] (c) Sub duplicate ratio of a : 9 = $\sqrt{a} : \sqrt{9}$, Compound Ratio (C.R.) = 8:15

Compound Ratio of 4:5 and sub duplicate ratio of a:9 is given by

$C.R = \frac{4}{5} \times \frac{\sqrt{a}}{\sqrt{9}}$

$\frac{8}{15} = \frac{4}{5} \times \frac{\sqrt{a}}{\sqrt{9}}$

$\sqrt{a} = \frac{8 \times 5 \times \sqrt{9}}{15 \times 4}$

$\sqrt{a} = \frac{8 \times 5 \times 3}{15 \times 4}$

$\sqrt{a} = 2$

On squaring $(\sqrt{a})^2 = 2^2$

$a = 4$

[55] (a) If $\log_2 x + \log_4 x = 6$

$\frac{\log x}{\log 2} + \frac{\log x}{\log 4} = 6$

$\frac{\log x}{\log 2} + \frac{\log x}{\log 2^2} = 6$

$\frac{\log x}{\log 2} + \frac{\log x}{2\log 2} = 6$

$\frac{\log x}{\log 2}\left[1 + \frac{1}{2}\right] = 6$

$\frac{\log x}{\log 2} \times \frac{3}{2} = 6$

$\frac{\log x}{\log 2} = 6 \times \frac{2}{3}$

$\frac{\log x}{\log 2} = 4$

$\log x = 4 \log 2$

$\log x = \log 2^4$

$x = 2^4$

$x = 16$

[56] (d) Given x varies inversely as square of y

i. e. $x \propto \frac{1}{y^2}$

$x = k\,\frac{1}{y^2}$

$x = \frac{k}{y^2}$................(1)

Given x = 1, y = 2 then

$1 = \frac{k}{(2)^2}$ k = 1 × 4 = 4

Now putting y = 6_1 k = 4 in equation (1)

$x = \frac{4}{6^2}$

$x = \frac{4}{36} = \frac{1}{9}$

[57] (b) $\frac{3^{n+1}+3^n}{3^{n+3}-3^{n+1}} = \frac{3^n.3^1+3^n}{3^n.3^3-3^n.3^1}$

$= \frac{3^n\,(3^1+1)}{3^n\,(3^3-3)}$

$= \frac{(3+1)}{(27-3)}$

$= \frac{4}{24}$

$= \frac{1}{6}$

[58] (c) Given $\log_x y = 100$ ....(1)

$\log_2 x = 10$......................(2)

Multiply eq (1) & (2)

$\log_x y\,.\,\log_2 x = 100 \times 10$

$\frac{\log y}{\log x} \times \frac{\log x}{\log 2} = 1{,}000$

log y = 1,000 log 2

log y = $\log 2^{1,000}$

y = $2^{1,000}$

[59] (a) If say a, b, c, d are in proportion they bear a common ratio that is $\frac{a}{b} = \frac{c}{d}$

Option (A) $\frac{6}{8}$ $\frac{5}{7}$

Option (B) $\frac{7}{3} = \frac{14}{6}$

Option (C) $\frac{18}{27} = \frac{12}{18}$

Option (D) $\frac{8}{6} = \frac{12}{9}$

[60] (b) If $x^1 (x)^{1/3} = (x^{1/3})^x$

$x^{1+1/3} = x^{\frac{1}{3}x}$

$x^{4/3} = x^{\frac{1}{3}x}$

on comparing

$\frac{4}{3} = \frac{x}{3}$

3x = 12 x = 4

[61] (d) Given

$\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca} = \frac{1}{abc}$

$\frac{c + a + b}{abc} = \frac{1}{abc}$

a + b + c = 1

taking log on both side

log (a + b + c) = log 1

log (a + b + c) = 0

[62] (a) Let two Nos. be x and y

Mean proportion between x and y is 18

So, x, 18, y are in proportion

x : 18 :: 18 : y

$\frac{x}{18} = \frac{18}{y}$

xy = 324

$x = \frac{324}{y}$ ________ (1)

If third proportion between x & y be 144

So, x, y, 144 are in proportion

$x : y :: y : 144$

$\frac{x}{y} = \frac{y}{144}$

$y^2 = 144x$ _________ (2)

Putting the value of x in equation (2)

$y^2 = 144 \times \frac{324}{y}$

$y^3 = 144 \times 324$

$y = \sqrt[3]{144 \times 324}$

$y = \sqrt[3]{3 \times 3 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3}$

$y = \sqrt[3]{6 \times 6 \times 6 \times 6 \times 6 \times 6}$

$y = 6 \times 6$

$y = 36$

Putting y = 36 in equation (1)

$x = \frac{324}{36} = 9$

$x = 9, y = 36$

[63] (a) Given

$$(\log_{\sqrt{x}} 2)^2 = \log_x 2$$

$$\left(\frac{\log 2}{\log \sqrt{x}}\right)^2 = \left(\frac{\log 2}{\log x}\right)$$

$$\left(\frac{\log 2}{\log x^{1/2}}\right)^2 = \frac{\log 2}{\log x}$$

$$\left(\frac{\log 2}{\frac{1}{2}\log x}\right)^2 = \frac{\log 2}{\log x}$$

$$\left(\frac{2\log 2}{\log x}\right)^2 = \left(\frac{\log 2}{\log x}\right)$$

$$4\left(\frac{\log 2}{\log x}\right)^2 = \left(\frac{\log 2}{\log x}\right)^1$$

$$4\ \frac{\log 2}{\log x} = 1$$

$$4 \log 2 = \log x$$

$$\log 2^4 = \log x$$

$$2^4 = x \quad \boxed{x = 16}$$

[64] (d) Mean Proportion = $\sqrt{24 \times 54}$

$= \sqrt{1296}$

$= 36$

[65] (c) The triplicate Ratio of 4 : 5 = $4^3 : 5^3$

= 64 : 125

[66] (a) If $\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c} = 0$

$a^{1/3} + b^{1/3} + c^{1/3} = 0$

$a^{1/3} + b^{1/3} = \quad c^{1/3}$ .... (i)

Cube on both side

$(a^{1/3} + b^{1/3})^3 = (\quad c^{1/3})^3$

$(a^{1/3})^3 + (b^{1/3})^3 + 3 \,.\, a^{1/3}.\, b^{1/3}\,(a^{1/3} + b^{1/3}) = \quad c$

$a + b + 3a^{1/3}.\, b^{1/3}.\,(\quad c^{1/3}) = \quad c$

$a + b \quad c\ 3a^{1/3}.\, b^{1/3}.\, c^{1/3} = \quad c$

$a + b + c = 3a^{1/3}.\, b^{1/3}.\, c^{1/3}$

$$\left(\frac{a+b+c}{3}\right) = \frac{\cancel{3}a^{1/3}.b^{1/3}.c^{1/3}}{\cancel{3}}$$

$$\left(\frac{a+b+c}{3}\right)^3 = (a^{1/3}.\, b^{1/3}.\, c^{1/3})^3 = abc$$

[67] (a) Since Ratio of three Number is 1 : 2 : 3

First No. = x

Second No. = 2x

Third No. = 3x

Sum of squares of numbers = 504

$(x)^2 + (2x)^2 + (3x)^2 = 504$

$x^2 + 4x^2 + 9x^2 = 504$

$14x^2 = 504$

$$x^2 = \frac{504}{14}$$

$x^2 = 36$

$x = 6$

First No. = x = 6
Second No. = 2x = 2 × 6 = 12
Third No. = 3x = 3 × 6 = 18

[68] (d) $\log_4 9 \cdot \log_3 2$

$= \frac{\log 9}{\log 4} \cdot \frac{\log 2}{\log 3}$

$= \frac{\log 3^2}{\log 2^2} \cdot \frac{\log 2}{\log 3}$

$= \frac{2\log 3}{2\log 2} \cdot \frac{\log 2}{\log 3}$

$= 1$

[69] (c) $(\log_y x \cdot \log_z y \cdot \log_x z)^3$

$= \left(\frac{\log x}{\log y} \cdot \frac{\log y}{\log z} \cdot \frac{\log z}{\log x}\right)^3$

$= (1)^3$

$= 1$

[70] (c) The sum of two No. = 80
First No. = x
Second No. = (80 x)
Product two No = x. (80 x)
$P = 80x \quad x^2$ .... (1)
w.r.f. (x)
$\frac{dp}{dx} = 80 \quad 2x$ .... (2)
$\frac{d^2p}{dx^2} = \quad 2$ .... (3)
For max/minima
$\frac{dp}{dx} = 0$
$80 \quad 2x = 0$
$2x = 80$
$x = 40$
x = 40 in equation (iii)

$\frac{d^2p}{dx^2} = \quad 2$ (Negative)

function is maximum at x = 40

Numbers are 40, (80   40)

= 40, 40

[71] (b) Given,

x : y = 2 : 3

Let x = 2k, y = 3k

(5x + 2y) : (3x   y)

$= \frac{(5x+2y)}{(3x-y)}$

$= \frac{5 \times 2k + 2 \times 3k}{3 \times 2k - 3k}$

$= \frac{10k + 6k}{6k - 3k}$

$= \frac{16k}{3k}$

= 16 : 3

[72] (b) If

$(25)^{150} = (25x)^{50}$

$25^{150} = 25^{50}. x^{50}$

$\frac{25^{150}}{25^{50}} = x^{50}$

$25^{100} = x^{50}$

$(5^2)^{100} = x^{50}$

$5^{200} = x^{50}$

$(5^4)^{50} = x^{50}$

$5^4 = x$

$x = 5^4$

[73] (c) $\left(\frac{y^a}{y^b}\right)^{a^2+ab+b^2} \quad \left(\frac{y^b}{y^c}\right)^{b^2+bc+c^2} . \left(\frac{y^c}{y^a}\right)^{c^2+ac+a^2}$

$= (y^{a-b})^{a^2+ab+b^2} . (y^{b-c})^{b^2+bc+c^2} . (y^{c-a})^{c^2+ac+a^2}$

$= y^{a^3-b^3} . y^{b^3-c^3} . y^{c^3-a^3}$

$= y^{a^3-b^3+b^3-c^3+c^3-a^3}$

$= y^0 = 1$

[74] (b) Let Salary of Q = 100

Salary of P = 100 25% of 100
= 100 25
= 75

Salary of R = 100 + 20% of 100
= 100 + 20
= 120

Ratio of salary of R and P = 120 : 75 = 8 : 5

[75] (b) If $x^2 + y^2 = 7xy$

$x^2 + y^2 + 2xy = 7xy + 2xy$

$(x + y)^2 = 9xy$

taking log on both side

$\log (x + y)^2 = \log 9xy$

$2 \log (x + y) = \log 9 + \log x + \log y$

$2 \log (x + y) = \log 3^2 + \log x + \log y$

$2 \log (x + y) = 2 \log 3 + \log x + \log y$

$2 \log (x + y) \quad 2 \log 3 = \log x + \log y$

$2 \left[\log \frac{(x+y)}{3}\right]$

$= \log x + \log y$

$\log \frac{(x+y)}{3} = \frac{1}{2} [\log x + \log y]$

[76] (b) A person has Assets worth = ₹ 1,48,200

Ratio of share of wife, son & daughter
= 3 : 2 : 1

Sum of Ratio = 3 + 2 + 1 = 6

Share of Son = $\frac{2}{6} \times 1,48,200$
= 49,400

[77] (c) If $x = \log_{24}12$, $y = \log_{36}24$ and $z = \log_{48}36$ then

XYZ + 1

$= \log_{24}12 \times \log_{36}24 \times \log_{48}36 + 1$

$= \frac{\log 12}{\log 24} \cdot \frac{\log 24}{\log 36} \cdot \frac{\log 36}{\log 48} + 1$

$= \frac{\log 12}{\log 48} + 1$

$= \frac{\log 12 + \log 48}{\log 48}$

$= \dfrac{\log(12 \times 48)}{\log 48}$

$= \dfrac{\log(576)}{\log 48}$

$= \dfrac{\log 24^2}{\log 48}$

$= \dfrac{2\log 24}{\log 48}$

$= 2.\dfrac{\log 24}{\log 36}.\dfrac{\log 36}{\log 48}$

$= 2.\log_{36}24 \,.\, \log_{48}36$

$= 2\ y\ z$

[78] (a) Given $\log x = a + b$, $\log y = a \quad b$

$\log\left(\dfrac{10x}{y^2}\right) = \log 10x \quad \log y^2$

$= \log 10 + \log x \quad 2\log y$

$= 1 + (a + b) \quad 2\ (a \quad b)$

$= 1 + a + b \quad 2a + 2b$

$= 1 \quad a + 3b$

[79] (b) If $x = 1 + \log_p qr$, $y = 1 + \log_q rp$, $z = 1 + \log_r pq$

$x = 1 + \dfrac{\log qr}{\log p}$

$x = \dfrac{\log p + \log qr}{\log p}$

$x = \dfrac{\log pqr}{\log p}$

$\dfrac{1}{x} = \dfrac{\log p}{\log pqr}$

Similarly

$\dfrac{1}{y} = \dfrac{\log q}{\log pqr}$

$\dfrac{1}{z} = \dfrac{\log r}{\log pqr}$

$\dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z} = \dfrac{\log p}{\log pqr} + \dfrac{\log q}{\log pqr} + \dfrac{\log r}{\log pqr}$

$= \frac{\log p + \log q + \log r}{\log pqr}$

$= \frac{\log pqr}{\log pqr}$

$= 1$

[80] (c) Ratio of the salary of a person in three months = 2 : 4 : 5

Let, Salary of Ist month = 2x

Salary of IInd month = 4x

Salary of IIIrd month = 5x

Given

(Salary of Product of last two months) (Salary of Product Ist two months) = 4,80,00,000

(4x.5x) (2x.4x) = 4,80,00,000

$20x^2$ $8x^2$ = 4,80,00,000

$12x^2$ = 4,80,00,000

x^2 = 40,00,000

x = 2,000

Salary of the person for second month = 4x = 4 × 2,000 = 8,000

[81] (a) Let SP of mixture is ₹ 100

Then Profit = 14.6% of 100

= 14.6

CP of mixture = (100 14.6)

= 85.4

If SP is ₹ 100 then CP = 85.4

If SP is ₹ 1 then CP = $\frac{85.4}{100}$

If SP is ₹ 17.60 then CP = $\frac{85.4}{100} \times 17.60$

= 15.0304

CP of the Mixture per kg = ₹ 15.0304

2nd difference = Profit by SP 1 kg of 2nd kind @ ₹ 15.0304

= 15.54 15.0304

= 0.5096

1st difference = ₹ 15.0304 13.84

= ₹ 1.1904

The Require Ratio = (2nd difference) : (1st difference)

= 0.5096 : 1.1904

= 3 : 7

[82] (d) If $p^x = q, q^y = r$ and $r^z = p^6$

$q = p^x, q^y = r$ and $r^z = p^6$

$(q^y)^z = p^6$

$[(p^x)^y]^z = p^6$

$p^{xyz} = p^6 = xyz = 6$

[83] (a) Log x = m + n and log y = m $\quad$ n

Then $\log\left(\frac{10x}{y^2}\right) = \log 10x \quad \log y^2$

$= \log 10 + \log x \quad 2 \log y$

$= 1 + \log x \quad 2 \log y$

$= 1 + (m + n) \quad 2 (m \quad n)$

$= 1 + m + n \quad 2m + 2n$

$= 3n \quad m + 1$

[84] (a) If $15(2p^2 \quad q^2) = 7pq$

$30p^2 \quad 15q^2 = 7pq$

$30p^2 \quad 7pq \quad 15q^2 = 0$

$30p^2 \quad 25pq + 18pq \quad 15q^2 = 0$

$5p(6p \quad 5q) + 3q(6p \quad 5q) = 0$

$(6p \quad 5q)(5p + 3q) = 0$

If $6p \quad 5q = 0$ and $5p + 3q = 0$

$6p = 5q$ $\qquad$ $5p = \quad 3q$

$\frac{p}{q} = \frac{5}{6} = p : q = 5 : 6$ $\qquad$ $\frac{p}{q} = \frac{-3}{5}$

(not possible)

[85] (b) The third proportion of 12,30

$c = \frac{b^2}{a} = \frac{(30)^2}{12} = \frac{900}{12} = 75$

The Mean proportion of 9,25

$b = \sqrt{ac} = \sqrt{9 \times 25} = \sqrt{225} = 15$

Ratio of third proportion of 12, 30

and Mean proportion of 9, 25 = 75:15

= 5:1

[86] (c) $\log_5 3 \times \log_3 4 \times \log_2 5$

$= \frac{\log 3}{\log 5} \times \frac{\log 4}{\log 3} \times \frac{\log 5}{\log 2}$

$= \frac{\log 4}{\log 2}$

$= \frac{\log 2^2}{\log 2}$

$= \frac{2 \log 2}{\log 2} = 2$

[87] (a) Let x to be added

Then (10 + x), (18 + x), (22 + x), (38 + x) are in prop.

Product of Extremes = Product of Mean

(10 + x) (38 + x) = (18 + x) (22 + x)

$380 + 10x + 38x + x^2 = 396 + 18x + 22x + x^2$

48x + 380 = 396 + 40x

48x - 40x = 396 - 380

8x = 16

x = 2

[88] (b) $\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n} = \frac{2^n + 2^n . 2^{-1}}{2^n . 2^1 - 2^n}$

$= \frac{2^n(1 + 2^{-1})}{2^n(2^1 - 1)}$

$= \frac{\left(\frac{1}{1} + \frac{1}{2}\right)}{(2-1)}$

$= \frac{\left(\frac{2+1}{2}\right)}{1}$

$= \left(\frac{3}{2}\right)$

[89] (b) The integral part of a logarithms is called **Characteristic** and the decimal part of a logarithm is called **mantissa**.

[90] (b) $\frac{x^2-(y-z)^2}{(x+z)^2-y^2} + \frac{y^2-(x-z)^2}{(x+y)^2-z^2} + \frac{z^2-(x-y)^2}{(y+z)^2-x^2}$

$= \frac{(x+y-z)(x-y+z)}{(x+z+y)(x+z-y)} + \frac{(y+x-z)(y-x+z)}{(x+y+z)(x+y-z)} + \frac{(z+x-y)(z-x+y)}{(y+z+x)(y+z-x)}$

$$= \frac{x+y-z}{x+y+z} + \frac{y+z-x}{x+y+z} + \frac{z+x-y}{x+y+z}$$

$$= \frac{x+y-z + y+z-x + z+x-y}{x+y+z}$$

$$= \frac{x+y+z}{x+y+z} = 1$$

[91] (d) Given x = 3y and y = $\frac{2}{3}$z

$$\frac{x}{y} = \frac{3}{1} \text{ and } \frac{y}{z} = \frac{2}{3}$$

x : y = 3 : 1 and y : z = 2 : 3

= 3 × 2 : 1 × 2

= 6 : 2

x : y : z = 6 : 2 : 3

[92] (c) If $\log_4 (x^2 + x) - \log_4 (x + 1) = 2$

$$\log_4 \left\{\frac{(x^2+x)}{(x+1)}\right\} = 2$$

$$\log_4 \left\{\frac{x(x+1)}{(x+1)}\right\} = 2$$

$\log_4 x = 2$

$x = 4^2$

x = 16

[93] (b)
$$\frac{1}{\log_3 60} + \frac{1}{\log_4 60} + \frac{1}{\log_5 60}$$

$$= \log_{60} 3 + \log_{60} 4 + \log_{60} 5 \qquad \left[\square \frac{1}{\log_a b} = \log_b a\right]$$

$$= \log_{60}(3 \times 4 \times 5)$$

$$= \log_{60} 60$$

= 1

[94] (c) If $3^x = 5^y = 75^z = k$ (let)

then $3^x = k$, $5^y = k$, $75^z = k$

$3 = k^{1/x}$, $5 = k^{1/y}$, $75 = k^{1/z}$

we know that

75 = 3 × 5 × 5

$$k^{\frac{1}{z}} = k^{\frac{1}{x}}.\,k^{\frac{1}{y}}.\,k^{\frac{1}{y}}$$

$$k^{\frac{1}{z}} = k^{\frac{1}{x}+\frac{1}{y}+\frac{1}{y}}$$

on comparing

$$\frac{1}{z} = \frac{1}{x}+\frac{1}{y}+\frac{1}{y}$$

$$\frac{1}{z} = \frac{1}{x}+\frac{2}{y}$$

$$\frac{1}{x}+\frac{2}{y}=\frac{1}{z}$$

[95] (c) If log 2 = 0.3010 and log 3 = 0.4771

then log 24 = log (2 × 2 × 2 × 3)

= log 2 + log 2 + log 2 + log 3

= 3 log 2 + log 3

= 3 × 0.3010 + 0.4771

= 0.9030 + 0.4771

= 1.3801

[96] (a) If abc = 2

$ab = \frac{2}{c} = 2c^{-1}$ $\quad a = \frac{2}{bc} = 2\,b^{-1}c^{-1}$

$bc = \frac{2}{a} = 2\,a^{-1}$ $\quad b = \frac{2}{ca} = 2\,c^{-1}a^{-1}$

$ca = \frac{2}{b} = 2\,b^{-1}$ $\quad c = \frac{2}{ab} = 2\,a^{-1}b^{-1}$

Given $\frac{1}{1+a+2b^{-1}} + \frac{1}{1+\frac{1}{2}b+c^{-1}} + \frac{1}{1+c+a^{-1}}$

$$= \frac{1}{1+a+2b^{-1}} + \frac{2b^{-1}}{2b^{-1}(1+\frac{1}{2}b+c^{-1})} + \frac{a}{a(1+c+a^{-1})}$$

$$= \frac{1}{(1+a+2b^{-1})} + \frac{2b^{-1}}{2b^{-1}+1+2b^{-1}c^{-1}} + \frac{a}{a+ac+1}$$

$$= \frac{1}{1+a+2b^{-1}} + \frac{2b^{-1}}{2b^{-1}+1+a} + \frac{a}{a+2b^{-1}+1}$$

$$= \frac{1+2b^{-1}+a}{1+a+2b^{-1}}$$

$$= 1$$

[97] (a)

Total no. of coins = 23

Ratio of ₹ 1 coin : ₹ 2 coins = 3 : 2

let No. of ₹ 1 coins = 3x

No. of ₹ 2 coins = 2x

No. of ₹ 5 coins = 23– 3x– 2x = 23– 5x

Total value of all coins = 43

$3x \times 1 + 2x \times 2 + (23 - 5x) 5 = 43$

$3x + 4x + 115 - 25x = 43$

$-18x = 43 - 115$

$-18x = -72$

$x = \frac{-72}{-18} = 4$

No. of ₹ 1 coins = 3x = 3 × 4 = 12

[98] (c) a : b = 2 : 3 $\frac{a}{b} = \frac{2}{3}$ ________(i)

b : c = 4 : 5 $\frac{b}{c} = \frac{4}{5}$ ________(ii)

c : d = 6 : 7 $\frac{c}{d} = \frac{6}{7}$ ________(iii)

Multiply equation (i) & (ii) & (iii)

$\frac{a}{b} \times \frac{b}{c} \times \frac{c}{d} = \frac{2}{3} \times \frac{4}{5} \times \frac{6}{7} = \frac{16}{35}$

[99] (b) $\log (1^3 + 2^3 + 3^3 + \text{------} + n^3)$

$= \log (\Sigma n^3)$

$= \log \left[\frac{n(n+1)}{2}\right]^2$

$= 2 \log \left[\frac{n(n+1)}{2}\right]$

$= 2 [\log n + \log (n + 1) - \log 2]$

$= 2 \log n + 2 \log (n + 1) - 2 \log 2$

[100] (b) If $a = \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}}$ and $b = \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}}$

$$a + b = \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}} + \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}}$$

$$= \frac{(\sqrt{6}+\sqrt{5})^2+(\sqrt{6}-\sqrt{5})^2}{(\sqrt{6}-\sqrt{5})(\sqrt{6}+\sqrt{5})}$$

$$= \frac{6+5+2\sqrt{30}+6+5-2\sqrt{30}}{(\sqrt{6})^2-(\sqrt{5})^2}$$

$$= \frac{22}{6-5} = \frac{22}{1} = 22$$

$$a \cdot b = \left(\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}}\right)\left(\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}}\right) = 1$$

$$\frac{1}{a^2} + \frac{1}{b^2} = \frac{b^2+a^2}{a^2 b^2} = \frac{(a+b)^2-2ab}{(ab)^2}$$

$$= \frac{(22)^2-2\times 1}{(1)^2} = \frac{484-2}{1} = 482$$